An Examination of the Fracturing of Rocks by Hydraulic Pressure

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ABSTRACT

Rock is a naturally occurring aggregate of minerals. A rock is a complicated material of many constituents and characteristics. Dependent on the magnitude and rate of change of applied force to alter the transient equilibrium conditions, rock will react in a complicated manner, at all times exhibiting all of the well-known characteristics of elasticity, plasticity, and viscosity within the parameters of test conditions.

All rocks do exhibit a property of aeolotropy; that is, inherent directional characteristics. These characteristics should be made use of in mining and kindred operations to design stable orientations of openings as well as to define optimum drilling and breaking directions. They may be defined by laboratory analyses together with underground observations and measurements.

As far as the writer has been able to determine little attention has been given to the rheology or the aeolotropy of rocks in the process of hydraulic fracturing.

It is herewith postulated that where directional characteristics are definable by a laboratory technique, from oriented drill core or oriented hand samples, they should be used to define preferred fracturing planes by strike and dip, and the azimuth or direction of preferred strain propagation, induced by an internally applied hydraulic pressure.

Research conducted in the Department of Mining Engineering at Queen's University, Kingston, Ontario, between September 1963 and April 1965 has now proven the hypothesis to be correct within the range of several suites of rock samples tested.

The paper is a severe condensation of a dissertation presented as a partial requirement for the degree of Doctor of Philosophy.

INTRODUCTION

As with most rheologically defined materials, rocks react differently in different environments to similar forces. Physical changes and deformations depend as much on the rate of change of stress as the magnitude of stress. In many instances the rate of change is more important. Under load, all real materials will flow if given time. If there is no time for flow the material behaves in a brittle manner.

Particular characteristics are developed in rocks during their periods of formation and during post formative geologic history. Whatever their mode of occurrence rocks are a record of their formation and changes. Where analyses are possible advantage should be taken of the natural inherent characteristics to help design economical functional structures.

A number of techniques of measurement and analysis have been developed. The fracturing of rock by hydraulic pressure has been examined by many authorities but few within the scope of available published literature have rationally used the actual rocks concerned in tests to examine directional characteristics or preferred strain propagation rates and directions.

In a number of investigations of the properties of rocks for mine design purposes and stress analysis problems, in which the writer has been associated, it has been demonstrated that there are definite principal strains in rocks which can be defined. It has been shown that preferred shear planes can be identified by a photoelastic analysis which can be used to improve design of optimum drilling and breaking directions for mining purposes. Definable principal strain directions are used to facilitate design directions of optimum stability for permanent mine openings.[1]

In hydraulic fracturing operations the same directional characteristics as outlined above, can be used to define preferred fracturing directions in the laboratory from oriented samples. It is mainly with this aim that this research has been conducted. Such a method could also be used to carry out the fracturing of rocks for other techniques of mining such as primary breaking.

The research is confined to the field of rock mechanics in which the aim is to define a preferred fracture plane and directions of strain propagation by analysis, and then to study the effects on the projected fracture planes by fracturing the samples.

The main intent is to show why and how rheological rock characteristics are used to define the azimuth and orientation of planes along which fracturing will occur when an internal stress is applied to a rock. By the method, the azimuth and orientation of potential fractures are determined before fracture testing and the directional preferences of drive are measured during fracturing testing of an unconfined oriented laboratory sample.

Significant laboratory and field research conducted over the past twenty years, which has been obtainable through many agencies is discussed as a prelude for the requirements for further research.

HISTORY

A critical examination of past research, and field application of theories of hydraulic fracturing shows no more or less a tendency to test by trial and error, than any other mining operation known. Usually rheological characteristics of rock materials have not been sufficiently considered. In laboratories a great deal of theoretical work has been carried out using homogeneous, isotropic, elastic materials, and using a great number of theoretical assumptions as a basis for calculations. These factors, probably fairly correct in singular application of the theories, and sometimes fairly correct in unique circumstances do not take into account the complexity of the reactions of multiple compound arrangements of minerals, and minerals in rocks.

In the field, much reliance is placed on results of ideal laboratory research and surface topographical and geological features, to plan a fracture. In the petroleum industry it has not been quite as important to be able to define directional control as compared with the problem of joining wells in solution mining, but the basic concepts are the same.

Many subsidiary factors expected to influence a fracture have been studied, and frequently accepted as principal criteria, but rarely, if ever, have all rock characteristics been considered. Many factors of influence have been studied. Porosity, permeability, fluid pressures, depth pressure gradients, thermal conductivity, slotting and jet piercing, areal spread, are some examples. Many other factors are subsidiary to the above.

Much emphasis has been placed on depth pressure gradients [Hubbert & Willis (1956)]. The classical theories of elasticity and plasticity of ideal solid materials have been applied to explain phenomena encountered. However, where some notable papers have been examined in detail they have been found to produce many contradictory conclusions. With some notable exceptions, mathematical expressions developed to explain rock behaviour under changing environments usually cover only a few factors and usually do not include the broad coverage provided in rheological concepts. Such treatments are common in work by Hubbert & Willis (1956), Timoshenko (1934), and others.

[1] C. I. M. Consultants Ltd., Kingston, Ontario (1961-1963).

Much reliance has been placed on surface topography [Heck (1960)]. This kind of evidence has been shown more recently to be not as reliable as previously believed [Hodgson (1957), (1964), Ritsema (1957), Adams (1957)]. The latter have much conclusive evidence that fracture orientation planes are not linearly related to depth. They also disagree that regional principal strains are tangential and radial to Earth's surface, and state that horizontal fractures are just as liable to occur at greater depth in the crust as vertical fractures. They (Hodgson, Ritsema, Adams) show that natural fractures in the crust, as earthquake phenomena, are nearly always transcurrent (strike slip) with horizontal movement. Recent measurements have shown that within the workable depth in Earth's crust horizontal stresses can be considerably greater than vertical. [Fidler, Proceedings of the International Conference "State of Stress in the Earth's Crust," Hast (1958) and Olsen (1957)].

Sonic, seismic, and associated methods of well logging have been used to determine rock characteristics in situ. Schlumberger (1958) and Halliburton (1964) to quote two of many, produce valuable information for use in measuring some characteristics of rocks. However, none to date have demonstrated a way of determining preferred directions in which a rock will fracture, in laboratory or field tests.

Geophysical researchers, already referred to, have produced much valuable information that should help in examining rocks but their measurements mainly deal with geotectonics and are not necessarily concerned with upper crustal regions in which mining operations are carried out, at least within our time.

Such a brief historical resume does not tell us much, except to give a broad outline of directions in which research has followed -- and does little to show the directions a fracture will follow.

PURPOSE OF FRACTURING

To create a fracture in rock at a specific depth below surface is the primary purpose of hydraulic fracturing. In the petroleum industry, it is required to improve contact of a hole with a natural reservoir of oil and increase the productivity of the oil well. The boundaries or limits of the reservoir with relation to a well may or may not be known. In solution mining practices a fracture must have a more precisely defined purpose; that of joining two or more wells through which fluids are circulated in a closed circuit.

At this date the ability to fracture a formation is not difficult to achieve. Many people have been engaged in research to make a design one of greater engineered accuracy and as economically as possible. Fracturing formations and resealing or cementing fractures has become a highly competitive and organized business. Excellent results have been achieved in some regions, mediocre results in others, and in some places outright failure -- either to fracture, or to increase productivity through a fracture.

To investigate the orientation and extent of a fracture has occupied a lot of man-hours in laboratory and field. At present there are too many methods in use to enumerate them here. It is of much more advantage to spend time on trying to determine which way a fracture will go and its planar orientation.

There appears to have been, up-to-date, an unwillingness, or inability of operators, to recognize rheological concepts of rock characteristics. The concepts of linear relationships of pressure with depth, and inferred orientations of principal strains, may be approximated globally although there is little absolute measurement to prove them. There are many investigations published to disprove them as general rules. It is not reasonable to make such assumptions for several square miles and at any depth, let alone assume it for an individual ore body or rock formation. It has been demonstrated a number of times on projects in which the writer has been engaged[2] that there can be sets of mutually perpendicular principal strains in any orientation. The conditions vary with the origin of formation and post formative geological history. There is no valid basis on which to assume that only horizontal or vertical stresses exist in all formations in

[2] Confidential Unpublished Reports.

Earth's crust. There are too many people concerned with these controversial factors to refer to, in this paper, but it may be safely stated that the camp is about equally divided.

CHARACTERISTIC RHEOLOGICAL CONCEPTS

Mathematical formulas have been developed to help explain some of the concepts [Newman, Maxwell and Nadai (1950), Zandman (1953)]. These ideas have been expanded to enable their applications to rock materials by Reiner (1960), Emery (1961).

In order that all basic concepts of elasticity, plasticity, and viscosity can be expressed on a time basis they are combined as follows:

<u>HOOKE SOLID</u> -- Perfect Elasticity

$$\text{Youngs Modulus} = \frac{\text{Stress}}{\text{Strain}}$$

$$E = \sigma / \epsilon$$

<u>NEWTON LIQUID</u> -- Perfect Liquid

Stress = Viscosity × Rate of Strain

$$\sigma = \eta \times \frac{d\epsilon}{dt}$$

<u>ST. VENANT SOLID</u> -- Solid Viscosity

Shearing Traction = Tangential Yield Stress

$$\tau = \xi . \tau .$$

<u>KELVIN SOLID</u> -- Elasticity and Solid Viscosity

Stress = (Young's Modulus × Strain) plus (Viscosity × Strain Rate)

$$\sigma = E\epsilon + \eta \frac{d\epsilon}{dt} \quad \underline{\text{in parallel}}$$

<u>MAXWELL LIQUID</u> -- Elasticity plus Newton Liquid

$$\text{Rate of Strain} = \frac{\text{Stress}}{\text{Viscosity}} \times \frac{\text{Rate of Stress}}{\text{Young's Modulus}}$$

$$\frac{d\epsilon}{dt} = \frac{\sigma}{\eta} + \frac{d\sigma}{dt} \cdot \frac{1}{E} \quad \underline{\text{in series}}$$

As far as the above forms of rheological expressions are concerned, the emphasis is on rate of change as being one of the most important factors.

Other important factors are:

1. variability of viscosity with grain size;
2. level of inherent strain in rocks;
3. granular packing pattern and cementing medium; and
4. granular distribution by size, strength, and orientation.

All of the above concepts together with porosity, permeability, chemical composition, and structural discontinuities make the study of rock characteristics a formidable one. A visible plane of weakness or preferred plane of shearing in rock in situ is difficult if not impossible to detect, particularly at depth. Seismic and X-ray diffraction work is helping to solve some of these problems and minimize the use of model and dimensional analyses. Photoelasticity as an instrument of measurement of rock behaviour is still in its infancy, but reasonable success using a reflected light technique has been achieved [Emery (1961), Pegler (1962), Smith (1962), Pegler (1965)].

During the past five years the writer has employed reflective photoelastic techniques in rock stress analysis and mine orientation design, and due to this experience considered that combining this with a newly developed instrumentation in present research, could help solve some of the problems encountered in hydraulic fracturing. Mainly, the determination of the orientation of preferred fracture planes and preferred directions of strain propagation.

PREPARATION OF SAMPLES

Two locations were selected representing two different types of rock.

Location 1 -- Potsdam Sandstone, map location 27 -- 1962, Geology of Gananoque, Published by the Geological Survey of Canada, Ottawa, 1962. Specific location is shown in Fig. 1.

Location 2 -- Marble, map location 28 -- 1959, Geology of Westport, Sheet 31 $\frac{C}{9}$, at grid 76°.00 plus 5.9 miles and 44°30' plus 3.15 miles. Specific location is shown on Fig. 2.

All samples were oriented and drilled, using a Winkie gas-powered drill with a six-inch diameter thin-walled diamond drill bit on a three-foot core barrel. In the laboratory the samples were trimmed to produce four oriented vertical sides, and where possible, one horizontal plane. The freshly-cut flat planes were instrumented with photoelastic material using an epoxy bonding cement containing a reflective medium.

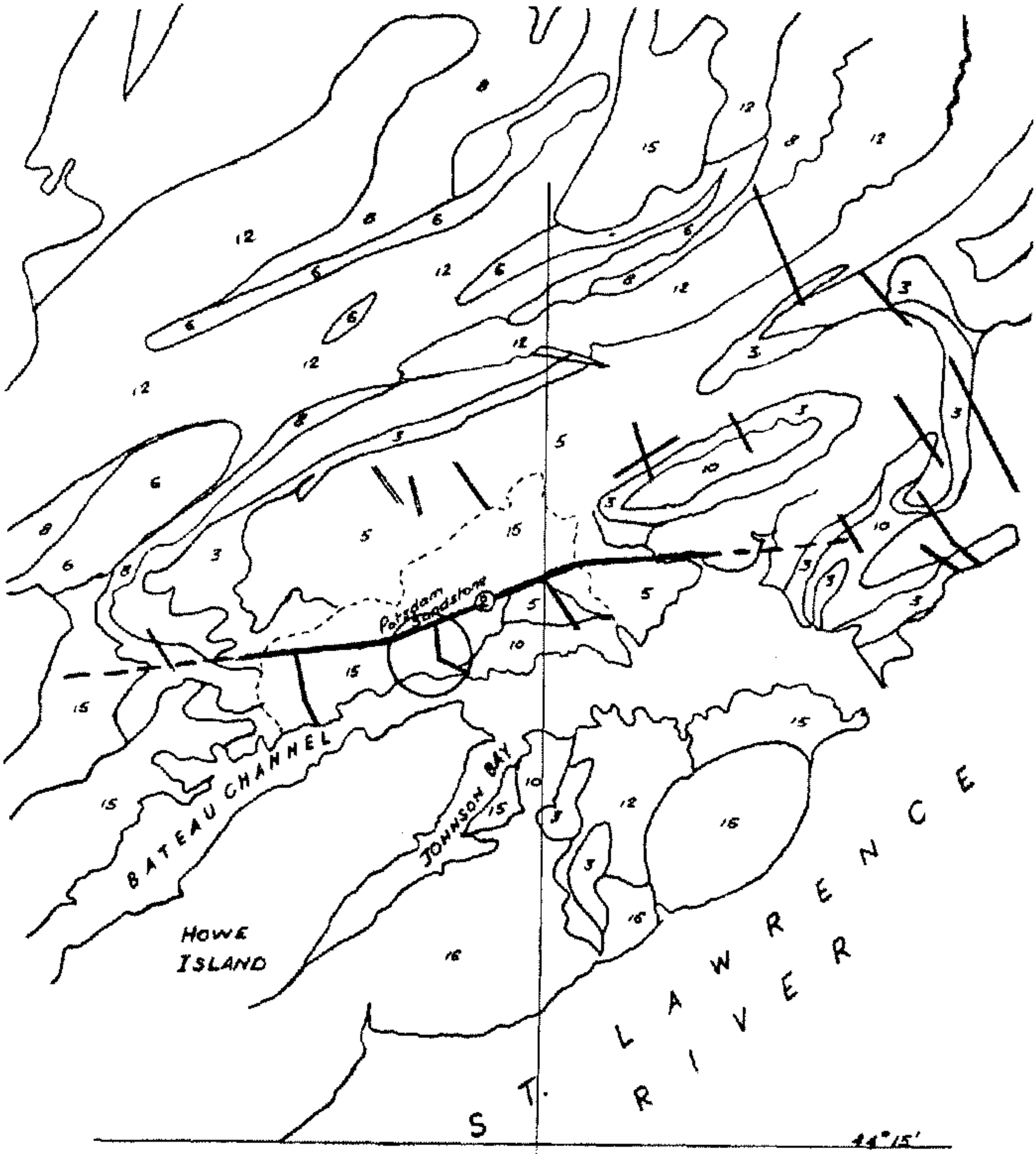
As energy relaxation occurred, strains were produced in the photoelastic material in such a pattern that principal strain directions and planes of preferred shear were identifiable. Inspection of the samples using white light through a polariscope enabled all characteristics to be marked and measured. On a day to day basis, points of highest shear strain, developed on the prepared faces, were observed. The readings are plotted on the time-relaxation energy graphs numbered Figs. 3, 4, 5, and 6 as examples.

Traces of shear are identifiable in relaxation as, at first, points of high shear strain producing a higher birefringence than the surrounding material. Secondly, with time, points of higher shear may show some alignment or may grow to join up, forming families of lines of constant colour density. The groups of lines are identifiable by viewing the sample through the polariscope using isochromatics, but they may be identified using anisoclinic view. When all sides have been observed results are plotted on a meridional stereonet and true strikes and dips obtained of preferred shear planes formed from the shear traces on the sample sides.

Principal strain directions are determined at points by alignment of the axes of the polariscope polaroids in the corresponding orientation of principal strains at the points. If a large number of points are observed over an area, a statistical average or directional lineation trend is obtained. The same effect is obtained by alignment of the polariscope axes with an overall isoclinic view of the same area. This is done on each instrumented side of a prepared sample and major and minor principal strain directions are identified by the direction of compensation of the polariscope analyzer. Those compatible strain directions in a three-dimensional view can also be plotted on the stereonet to produce the resultant field force direction, from the three components determined on three orthogonal faces.

The above brief description outlines the basis of prefracture analysis as far as directional characteristics are concerned. The magnitudes of shears, observed by a method of compensation of the polariscope and calculated by the use of basic photoelastic formulae, are at present, relative. The shear strain readings in micro inches per inch may be converted to figures expressing shear stress for the rock if some reference constant is used. For this some figure of comparison is required to be used between the known Young's Modulus and Poisson's Ratio of the photoelastic plastic and these calculated figures for the rock material.

As the predicted fracture information became available on each sample, the measurements were plotted on stereonets and all information filed in sample order. As each sample was test fractured results were entered in the respective sample files and immediately correlated with the individual sample predictions. Statistical evaluations were made after testing had been completed on all samples.

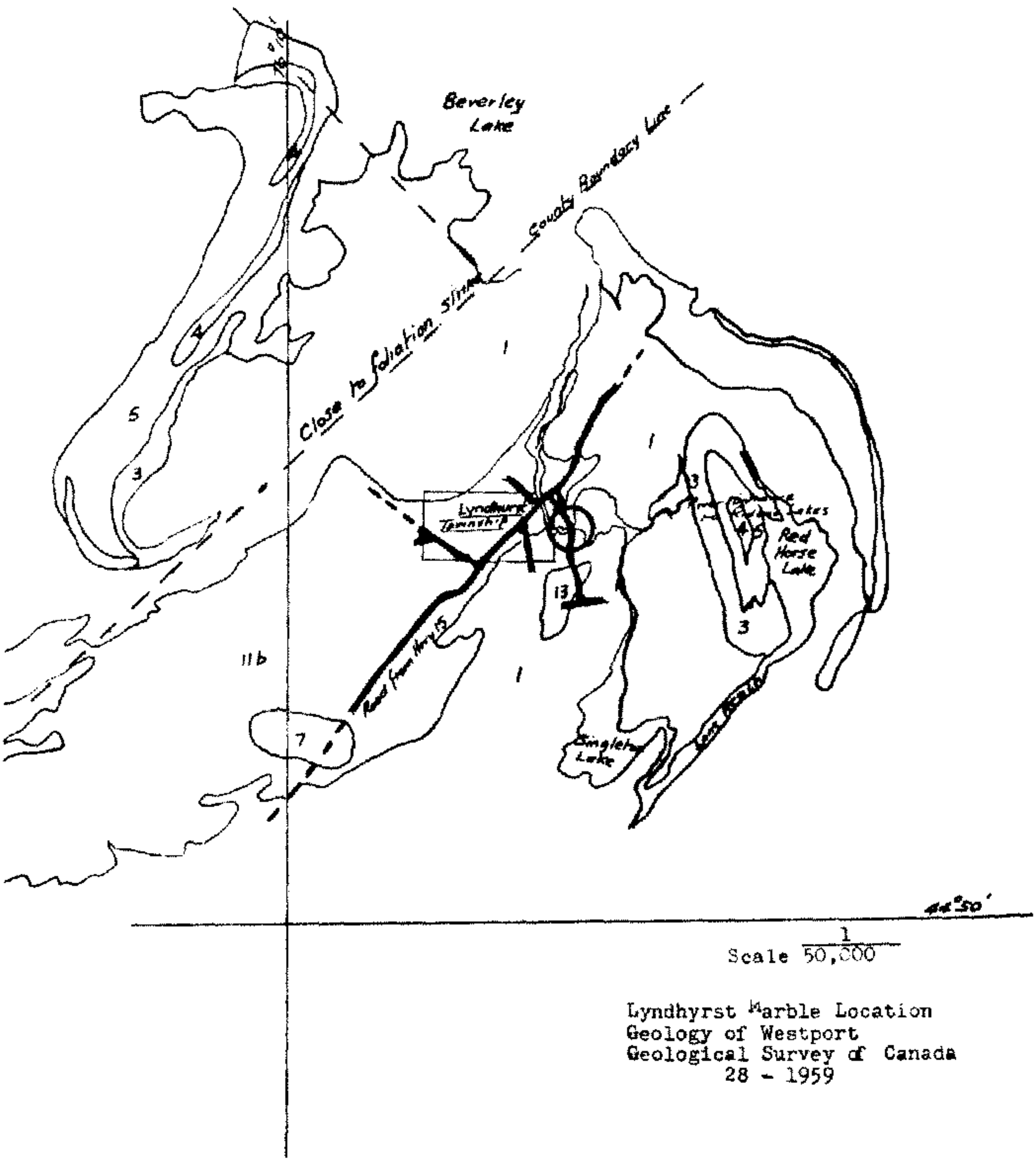


Scale 1 inch to 1 mile
= $\frac{1}{63,360}$

76°15'

Potsdam Sandstone Location
Geology of Gananoque Plan
Geological Survey of Canada
27 - 195[illegible]

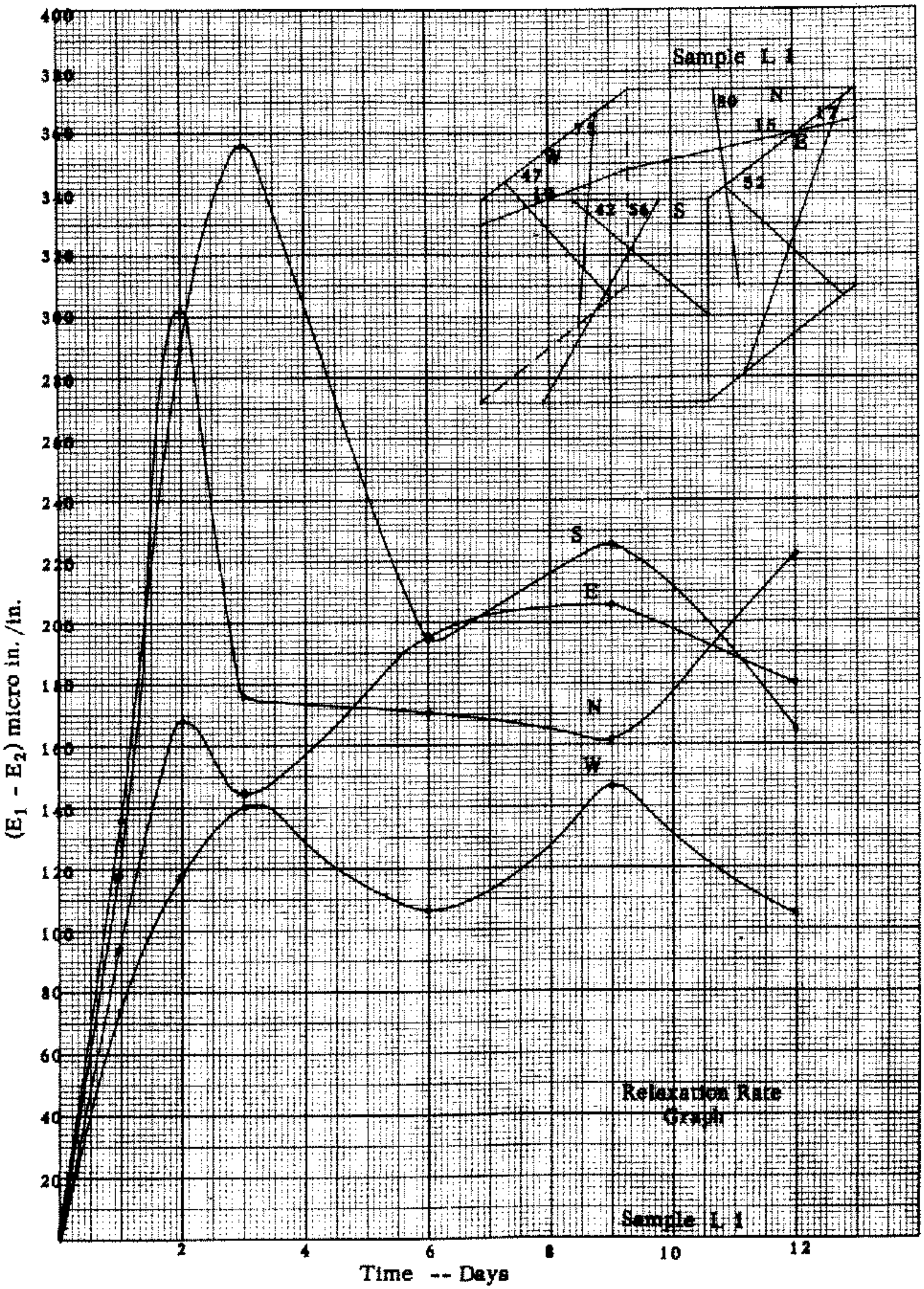
Figure 1



Scale $\frac{1}{50,000}$

Lyndhyrst Marble Location
Geology of Westport
Geological Survey of Canada
28 - 1959

Figure 2



Sample L 1
N
W
S
E
Relaxation Rate Graph
Sample L 1
(E1 - E2) micro in. /in.
Time -- Days

Figure 3

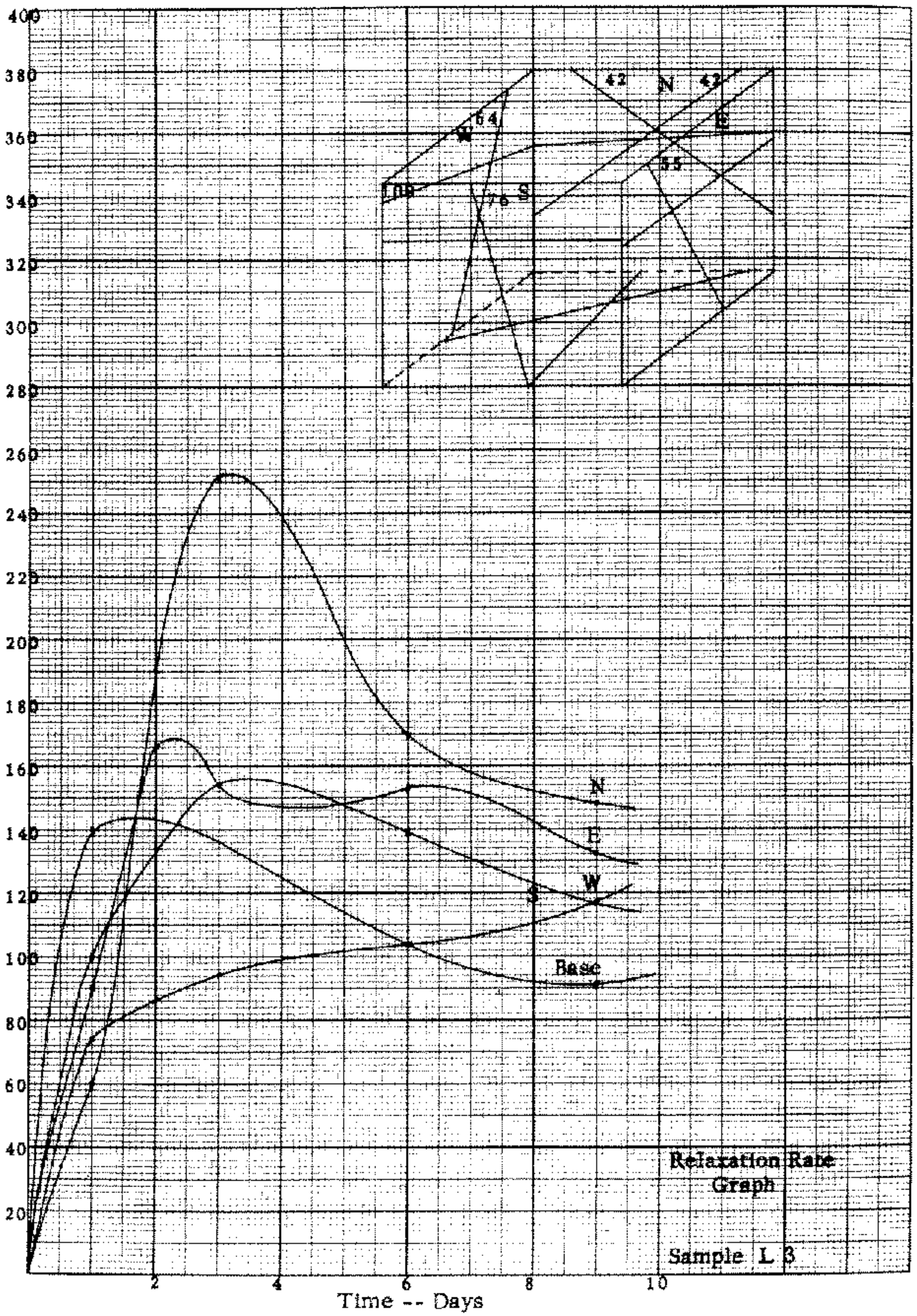

400
380
360
340
320
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
2
4
6
8
10
Time -- Days
N
42
W
64
E
55
S
76
108
N
E
W
S
Base
Relaxation Rate
Graph
Sample L 3

Figure 4

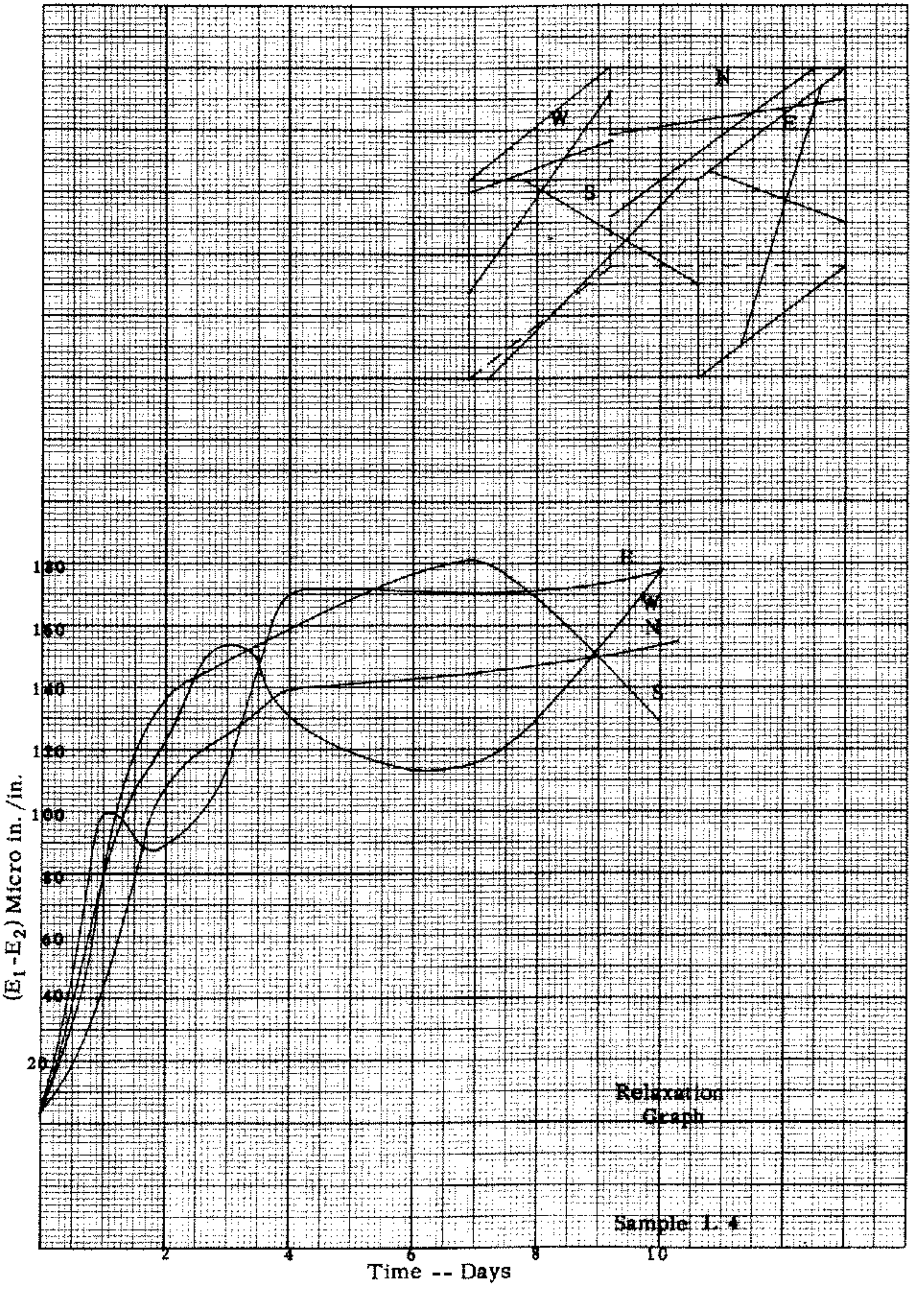
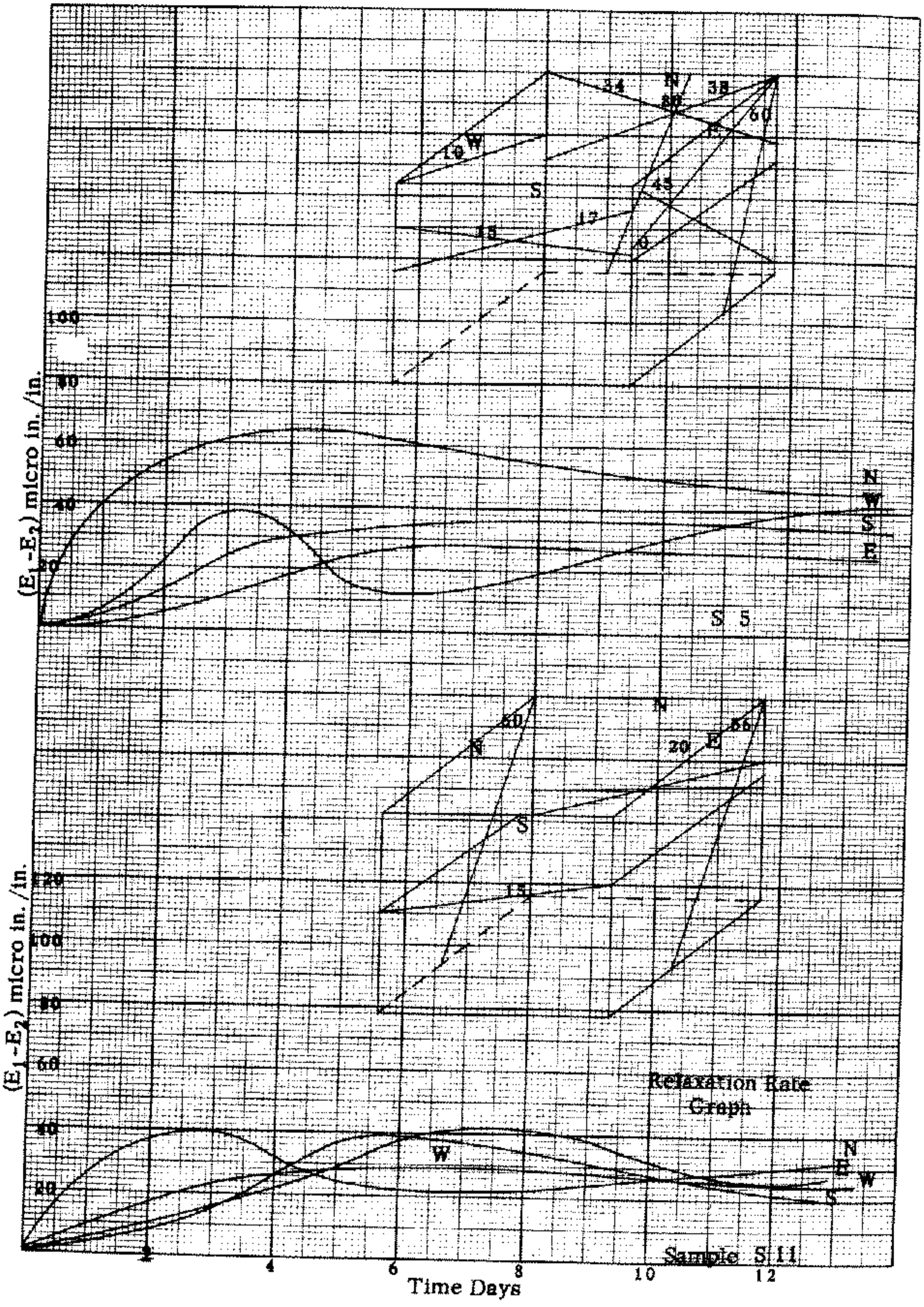

N
W
E
S
E
W
N
S
180
160
140
120
100
80
60
40
20
2
4
6
8
10
Relaxation Graph
Sample 1. 4
$(E_1 - E_2)$ Micro in. /in.
Time -- Days

Figure 5



(E1 -E2) micro in. /in.
100
80
60
40
20
N
W
S
E
S 5
(E1 -E2) micro in. /in.
120
100
80
60
40
20
Relaxation Rate
Graph
N
E
W
S
Sample S 11
2
4
6
8
10
12
Time Days

Figure 3

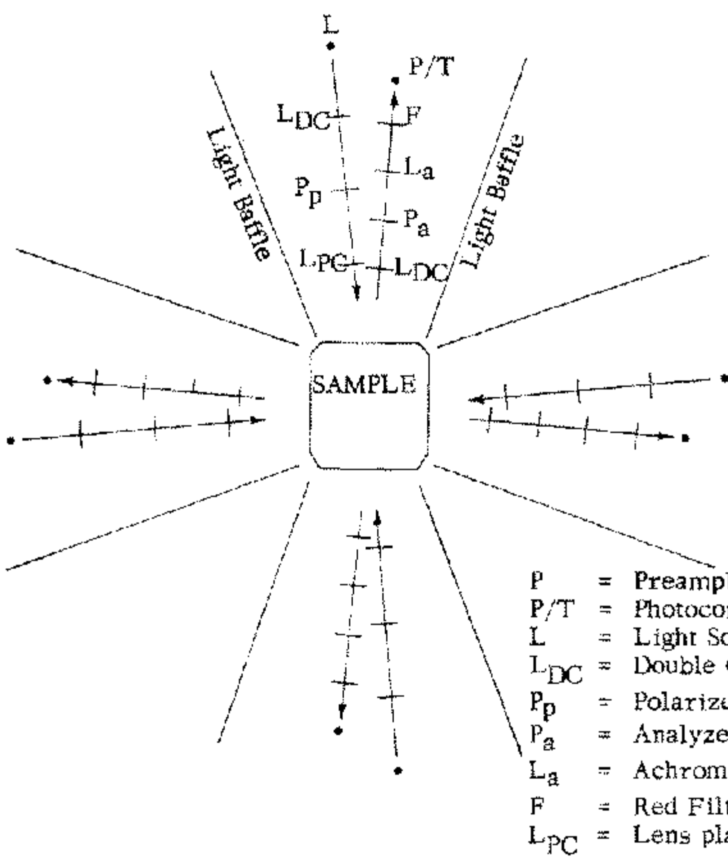
INSTRUMENTATION

Birefringence changes, transferred by a suitable polarizing optical system Fig. 7 and 7A register as resistance changes on a photoconductive cell. These effects are amplified by a specially designed amplifier and transmitted to Sanborn continuous chart recorders. This arrangement is set up to monitor all four vertical sides of photoelastically instrumented samples. By the optical system the total two-inch diameter illuminated view is integrated down to cover a one half-inch diameter circle. This area is covered by the sensitized portion of a photoconductive tube. Therefore, it is the overall birefringence change which is recorded, not the change in any single shear line.

The Sanborn recorder was chosen for its range of operation and versatility. The system was designed so that the overall birefringence change, which was required to be monitored, would be recorded by the deflection of the recorder styles within the usable chart paper width.

Cameras were mounted on each of the four recording sections with suitably oriented polaroid filters placed across each lens. Color film was used, and all four cameras were coupled to a common shutter release to simultaneously record effects. Unfortunately, all four views were not always available as it was not possible to judge the magnitude of birefringence changes in time to readjust exposures. Variable changes produced different colour temperature ratios, and frequently caused overexposure of one or more recording films. Some comparative views are shown in Figs. 8, 9, 10, and 11.

Better resolution of the reflected beam to photoconductive tubes was obtained by placing red filters directly in front of the tubes. It is possible that better photographic results might have been obtained if the cameras had also been fitted with comparable filters for monochromatic light.



Nomenclature

- P = Preamplifier
- P/T = Photoconductive tube
- L = Light Source
- L_{DC} = Double Convex lens
- P_p = Polarizer
- P_a = Analyzer
- L_a = Achromatic lens
- F = Red Filter
- L_{PC} = Lens plano convex

Figure 7. Optical System.

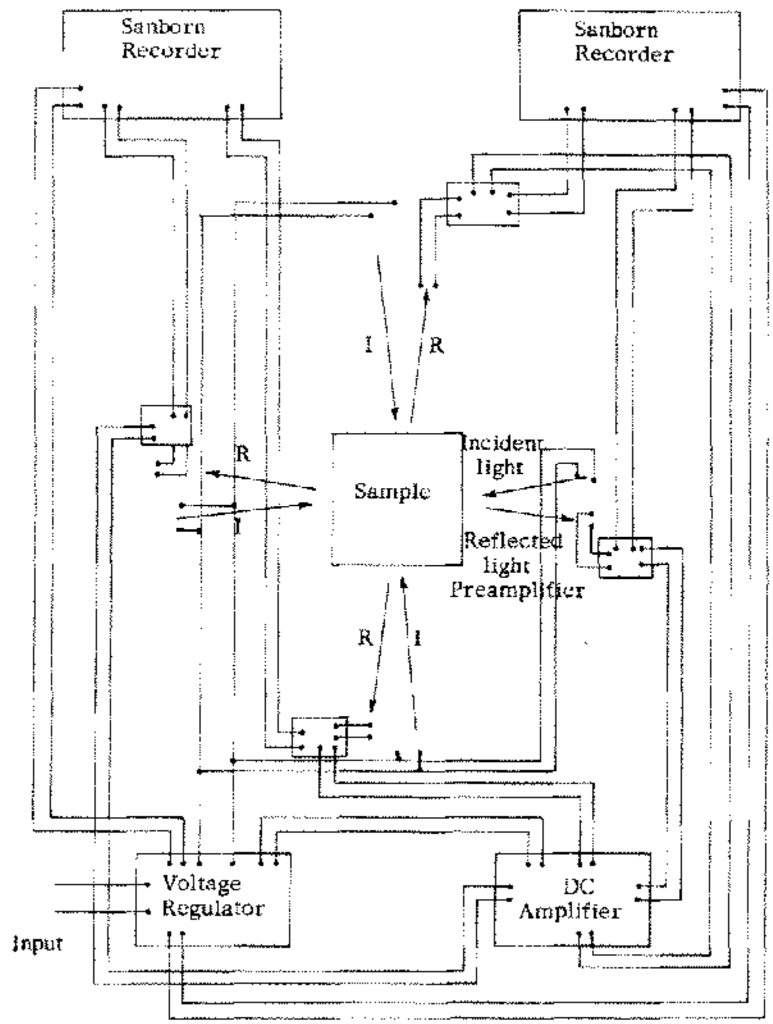


Figure 7A. Diagrammatic Circuit.

Pressure Tests and Measurements

Each sample was fitted with a steel casing, which was perforated at a fracturing horizon. The casing was bonded in a hole drilled in the sample centre normal to the horizontal plane. The bonding cement used was a high strength epoxy resin. Pressures of up to 2,000 p.s.i. caused no ejection or measureable movement of casings.

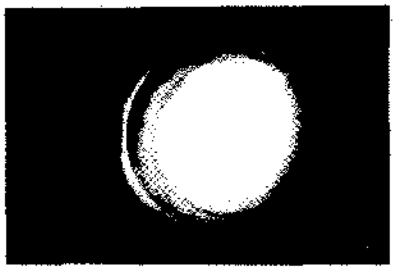
Hydraulic fluid was injected directly into the samples by an electrically controlled constant pressure hydraulic pump. An auxiliary gauge was fitted in the hydraulic line as a check on the pump gauge.

At the commencement of a fracture test and when a readable pressure was recorded on the auxiliary gauge, the Sanborn recorders were simultaneously started. A second time cycle along the edge of each twin chart accurately recorded the time of operation. The method of pressure change was as follows:

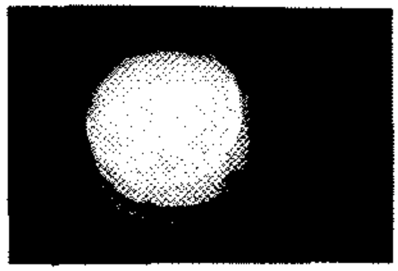
A pressure of 200 p.s.i. was permitted to be slowly attained by careful control of the pump by-pass valve. Pressure was held for two and one half minutes at each increment of 100 p.s.i. Pressure was raised by 100 p.s.i. in one half minute at the end of each holding period. Therefore an increase increment of 100 p.s.i. was made every three minutes. Jolt or impact pressure increases were avoided and careful control maintained. The recorder chart speed was maintained at 1 mm. per second and stylus amplification (a function of the recorder) held at one half volt per cm. of chart width.

ZERO LOAD

AT FRACTURE LOAD



NORTH FACE



NORTH FACE

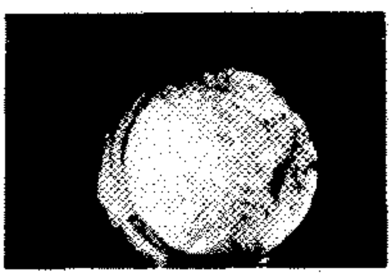
SOUTH FACE

SOUTH FACE

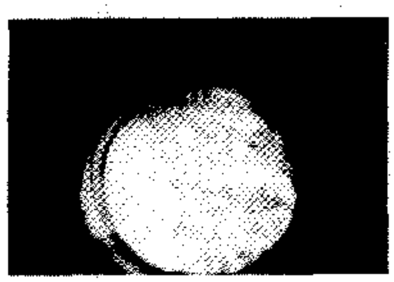
Figure 8. Potsdam Sandstone S11.

ZERO LOAD

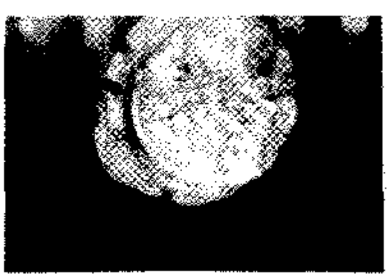
AT FRACTURE LOAD



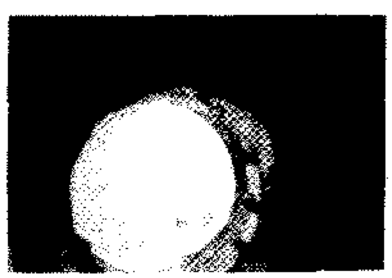
EAST FACE



EAST FACE



WEST FACE



WEST FACE

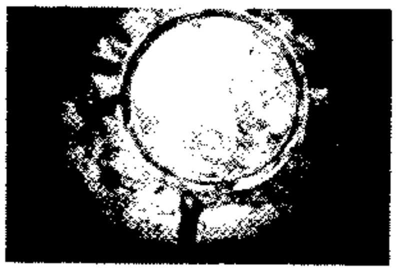
Figure 9. Potsdam Sandstone S11.

ZERO LOAD

AT FRACTURE LOAD

NORTH FACE

NORTH FACE



SOUTH FACE

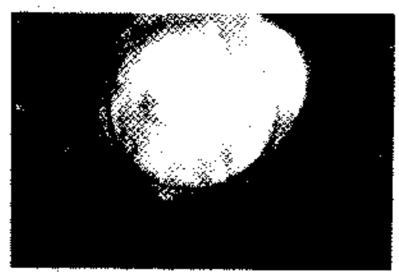
SOUTH FACE

Figure 10. Lyndhurst Marble L4.

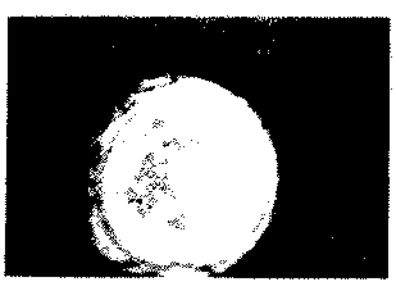
ZERO LOAD

AT FRACTURE LOAD

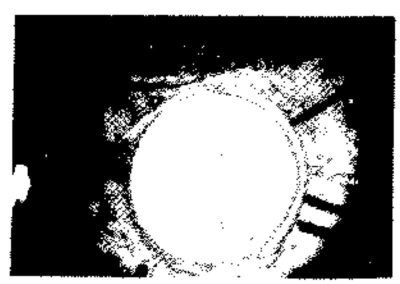
EAST FACE



EAST FACE



WEST FACE



WEST FACE

Figure 11. Lyndhurst Marble L4.

Recorder Calibration

Each measuring unit was required to be synchronized with its respective recorder channel. Each photoelastically instrumented face of the rock sample would not be expected to be of exactly the same strain (birefringence) output, even at zero load. The phototube amplifiers were fitted with adjustable zeroing rheostats, and a zeroing technique developed. By each channel being zeroed in with the registered resistance on each phototube, a measurement was obtained of the relative channels output at zero load. Apart from this, the zero control was necessary in order that the maximum stylus deflections could be compared after fracture testing (see Fig. 11A).

Immediately after a strain change occurred it was registered by a deviation of the corresponding Sanborn stylus. Differential strain change-rates were obtained by comparing the change points with respect to the time scales. Magnitudes of overall strain increase were measurable as long as the chart limits were not exceeded. See Figs. 12 and 13.

Amplification and Power Control

In order to determine the range of birefringence changes to be expected in fracture testing, specific amplification had to be designed. Preliminary testing showed that over the expected change range, a resistance change from $4.8 \times 10^3\ \Omega$ to $3.00 \times 10^3\ \Omega$ could be measured on a voltmeter. This occurred between zero load effect and the formation of four photoelastic fringe

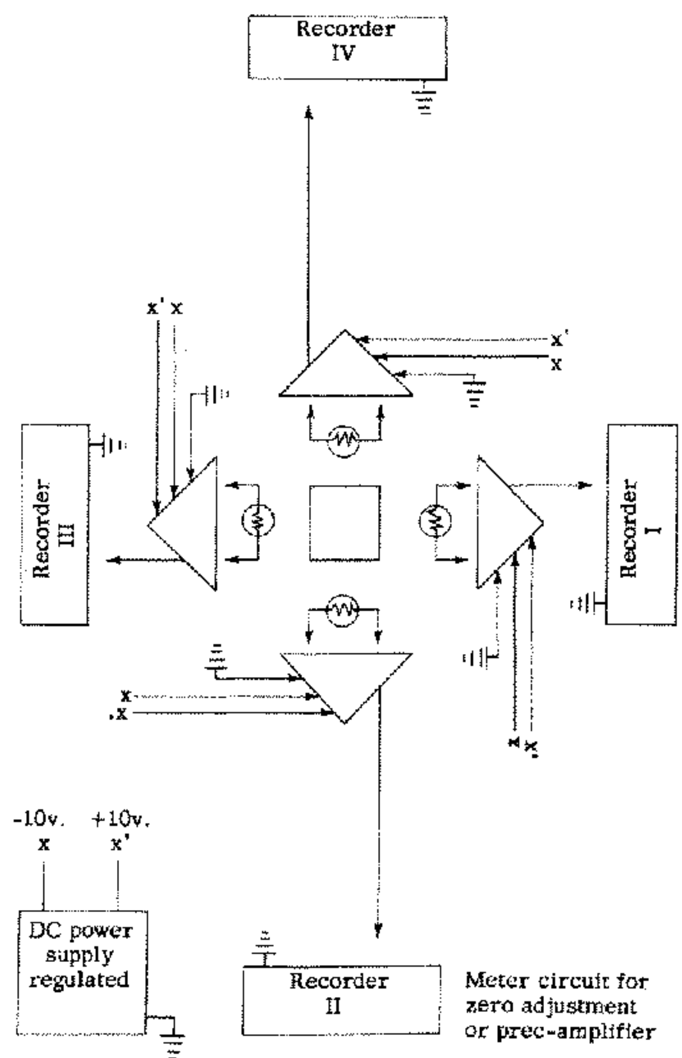


Figure 11A

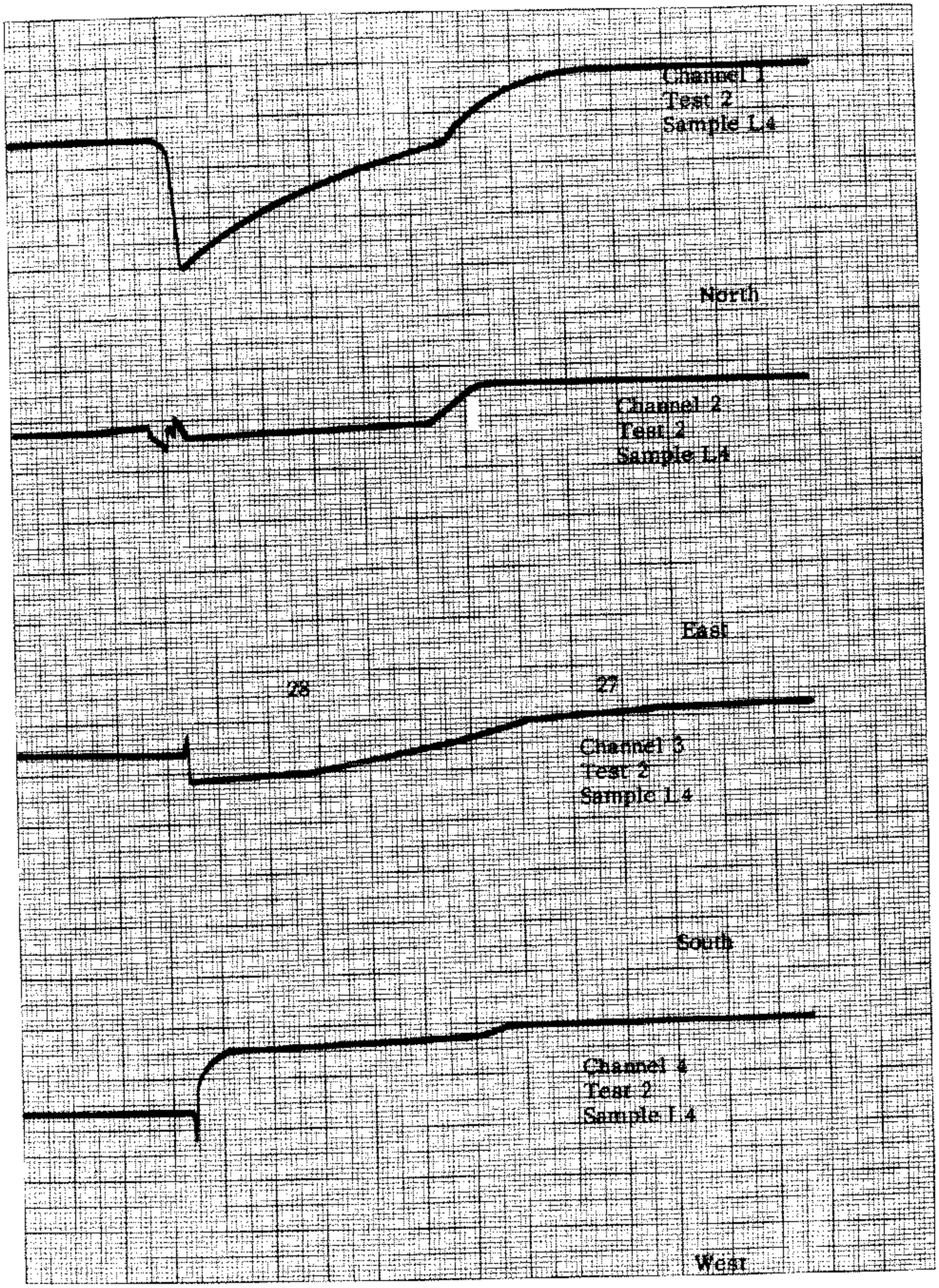

Channel 1
Test 2
Sample L4
North
Channel 2
Test 2
Sample L4
East
28
27
Channel 3
Test 2
Sample L4
South
Channel 4
Test 2
Sample L4
West

Figure 12

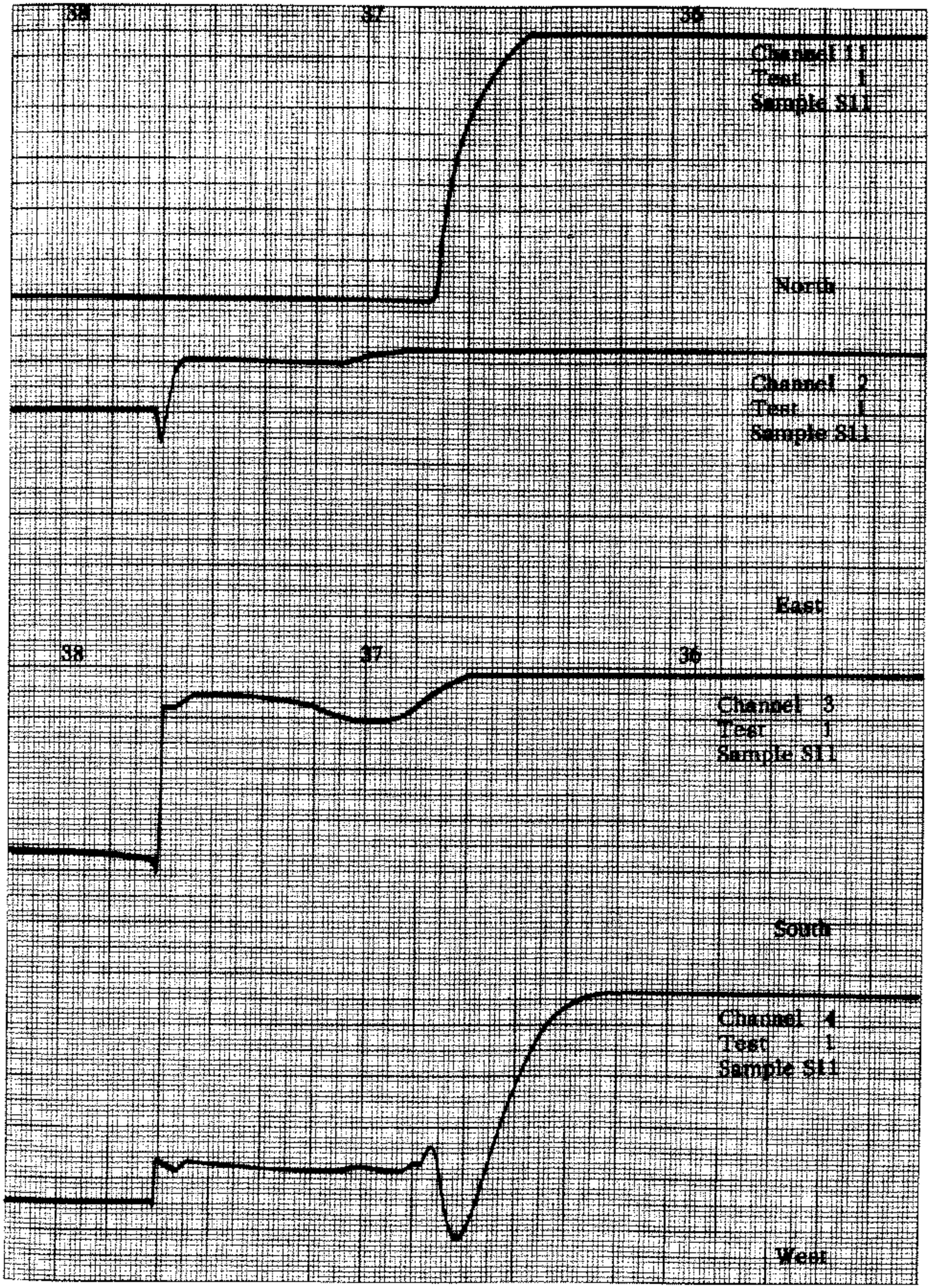

38
37
36
Channel 1
Test 1
Sample S11
North
Channel 2
Test 1
Sample S11
East
38
37
36
Channel 3
Test 1
Sample S11
South
Channel 4
Test 1
Sample S11
West

Figure 13

orders (Appendix). See Fig. 14. A gradual strain increase on the photoelastic material caused a gradual dampening out of oscillations registered on the recorder. When an overall yellow to orange birefringence colour predominated, the phototube output was maximum -- about 7,300 A• units. Changes from there were decreased, total resistance became greater and variations decreased.

From the determined range of operation, an amplifier circuit was designed for preliminary testing, which when combined with modifications, shielded conductors, and heat sinks, finally produced a more sophisticated amplifier system using primary and secondary solid state transistorized units. The use of a voltage regulator protected the input to all equipment from voltage fluctuations.

From a Sola voltage regulator power is fed into a D.C. amplifier. A four-wire shielded conductor conveys the amplifier output to the phototube amplifiers. Resistance changes across the photoconductive tubes are thus amplified to a workable and controllable range before transmission to the recorders.

From the photoelastic method of prefracture analysis the following characteristics were determined:

1. Preferred Shear Planes -- The planes along which rock will prefer to break when stresses are applied to change the condition of equilibrium.
2. Principal Strain Directions -- The directions of statistical principal strains, E_1 and E_2 in each plane, measured as components of the existing major field force.
3. Points of High Shear Strain -- Along the traces of shear planes on each face, high strain points were examined daily. The changes occurring with time are plotted on the accompanying graphs (Figs. 3 to 6) to show the energy relaxation rates inter, and intra granularly in the respective planes of the samples.

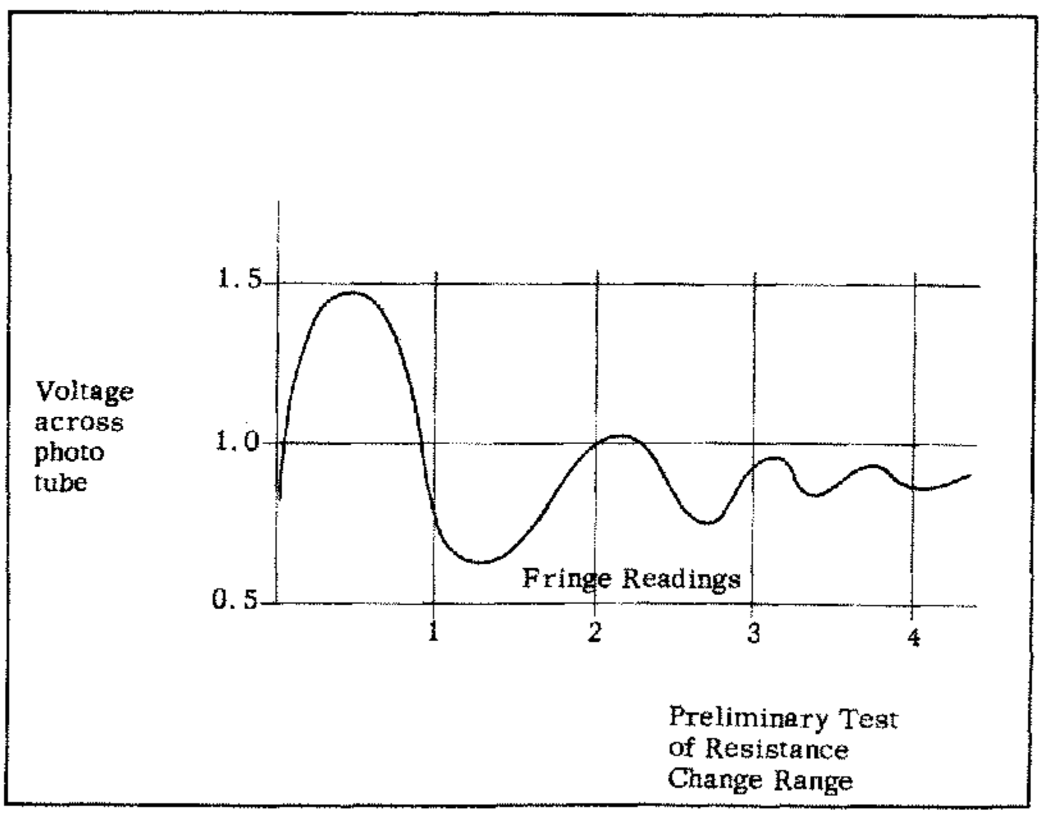


Figure 14. Preliminary Test of Resistance Change Range.

The observations of shear and strain directions were plotted on a meridional stereonet (Figs. 15 to 17). True strikes and dips were obtained of the prefracture shear planes and the direction and inclination of the major field force was determined. Measurements of the actual fractures were also plotted on the stereonet. They were superposed on the plots of prefracture characteristics. All figures of pre- and postfracture measurements were entered in the Tables 1 to 7. They were all referred to a magnetic meridian. The local variation between true and magnetic is 11°.

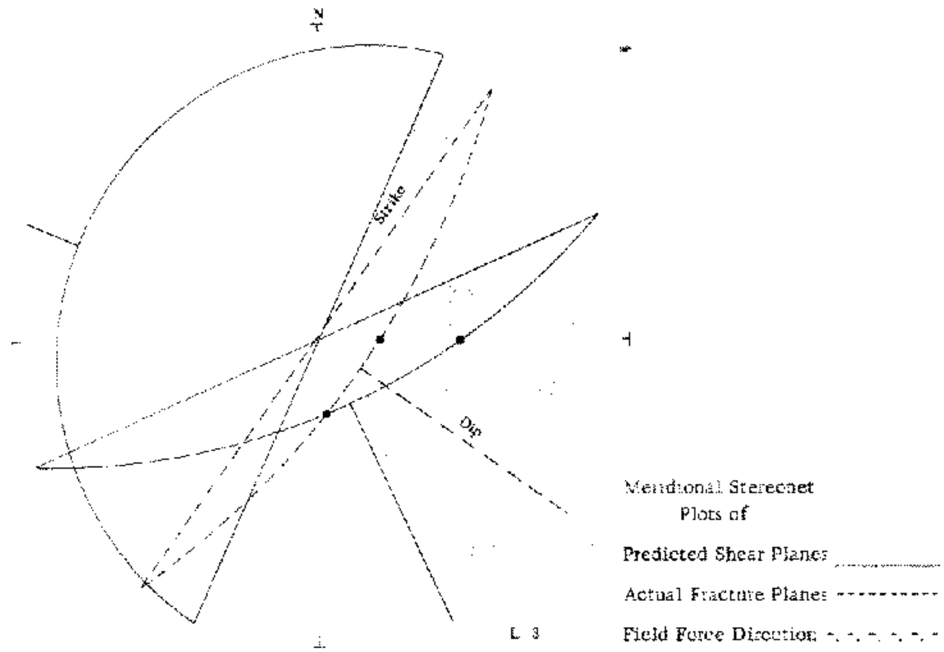


Figure 15

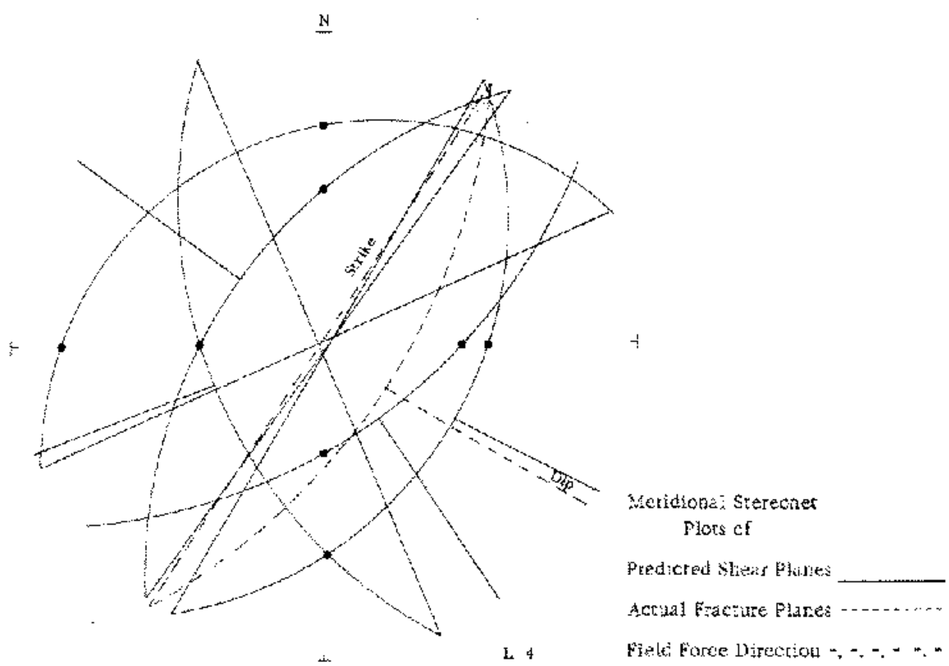
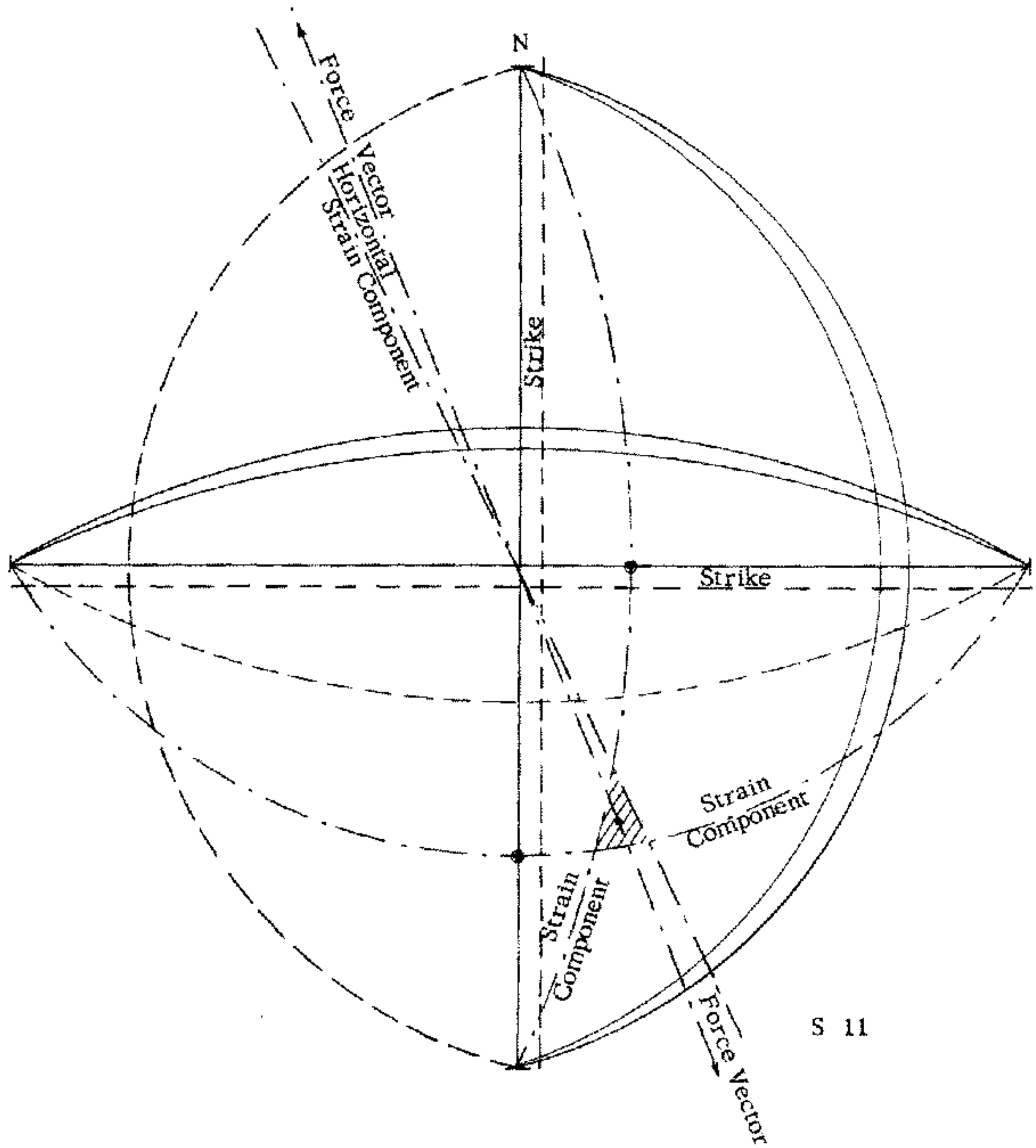


Figure 16

To accurately record the azimuth in which strain propagated, a time scale was used along the right-hand edge of the Sanborn recorder charts. After the completion of each test, the strip charts were inspected together with recorded deflection and a measurement was made of the exact time of strain commencement on each oriented sample plane.



Meridional Stereonet
Plots of

Predicted Shear Planes ________

Actual Fracture Planes ---------

Field Force Direction -.-.-.-.-

Figure 17

TABLE 1

POTSDAM SANDSTONE

Sample Number	Fracture				Force Vector	
	Predicted		Break Plane			
	Strike° Magnitude	Dip°	Strike° Magnitude	Dip°	Direction°	Inclination°
S4	032	48SE	000	10E	158	42
	090	27S	060	33S		
	052	52NW				
	154	11SW	180	10E		
S5	035	18SE	065	40SSE		
	000	39W	000 } strain { horizontal only			
			000	28W		
S6	060	6SE	072	70SE	163	56
S7	010	42E	017	45E		
	100	42N				
	130	14NE				
S11	090	58S	090	10N	159	34
	000	17W	000	7W		
S12B	106	63SW	042	vert.		
	152	32SW				
S13	105	16SSW	139	60NE		
	087	50N	050	8SE		
S15	121	59NE	154	22WSW		
	128	36SW	162	70WSW		
S16	090	33S	090	40S	150	42
	130	55SW				
	090	26N				
S17	000	20W	010	65W	124	34
	017	38E	100	67S		

TABLE 2

LYNDHURST MARBLE

Sample Number	Fracture				Force Vector	
	Predicted		Break Plane			
	Strike° Magnitude	Dip°	Strike° Magnitude	Dip°	Direction°	Inclination°
L1	034° 052° 130°	81°SE 67°NW 55°NE	000°	77°E	164°	47°
L2	010° 126°	75°E 14°NE	028°	75°E	142°	75°
L3	023° 067°	10°WNW 65°SSE	035°	70°SE	158°	40°
L4	030° 038° 055° 066°	40°ESE 53°NW 59°SE 22°WNW	033°	64°ESE	130°	42°
L5	132° 129° 049° 060°	43°NE 60°SW 71°SE 43°NW	145°	73°NE		
L6	032° 060° 133° 142° 150°	69°SE 64°SE 60°SW 70°SW 32°NE	045°	vert.		
L7	036° 046° 053°	85°SE 48°NW 16°SE	030°	72°SE	?	?
L8	000° 134°	66°W 24°NE	013° 129 } strain { only	vert. 24°NE		
	358° 112° 020°	72°SE 70°SSW 15°ESE				
L9	047° 050°	54°SE 70°NW	020°	63°ESE	048°	72°

TABLE 3

LYNDHURST MARBLE

Sample Number	Direction Line of Break Plane		Dip	Preferred Quadrant of Fracture Propagation		Remarks
	Primary	Secondary	Degree	Primary	Secondary	
L9	020°m		63°ESE	North	East	
L8	013	314°m	85°NW 24°NE	West	South	
L7	030		72°SE	South	West	
L6	045		vert.	South	West	
L5	145		73°ENE	West	South	
L4	033		64°ESE	South	North }	unusually low load
L3	035		70°SE	North	South	
L2	020		75°E	South	West	
L1	000		77°E	North	South	
POTSDAM SANDSTONE						
S 17	100°m	010°m	67°S 65°W	East	North	
S 16	090		38°S	East	North	
S 15	342	334	70°WSW 20°WSW	North	East	
S 13	139	050	60°NE 6°SE	East	N, S, W }	channels reacted simultaneously
S 12B	215		vert.	West	North	
S 11	000	270	7°W 10°N	West	North	
S 7	017		45°E	East	North South }	simultaneously
S 6	066		72°SE	direct.	simult.	
S 5	245	000	40°SSE 30°W	West	North	
S 4	250	000	33°S 10°E	West	North	

TABLE 4

CORRELATION OF STRAIN ENERGY RELAXATION
WITH STRAIN PROPAGATION SECTORS

Lyndhurst Marble:

Sample Number	Vertical Planes Showing Highest Strain Energy in Relaxation		Sectors of Preferred Strain Energy Relief Under Stress	
	Primary	Secondary	Primary	Secondary
L1	North	South	North	South
L2	South	West	West	S & N equal
L3	North	South	North	East
L4	South	North	South but all fairly even	
L5	West	South	South	North
L6	South	West	South	West
L7	South	West	South	West
L8	West	South	West	South
L9	North	East	West	N, E, S } high and equal

Potsdam Sandstone:

Sample Number	Primary	Secondary	Primary	Secondary
S 4	West	East	West	North
S 5	North	West	West	North
S 6	South	E, W equal	All equal	
S 7	North	East	East	N, S } simultaneous
S 11	North	West	West	North
S 12B	South	North	West	North
S 13	North	West	East	N, S, W } simultaneous
S 15	North	East	North	East
S 16	East	North	East	North
S 17	North	E, S equal	East	North

Figures 18 and 19 show the azimuth relationship of preferred strain directions. A double arrowhead shows a primary direction; single shows the secondary direction. An arrow at each end of a line indicates approximately equal propagation. The plots are true azimuth.

TABLE 5

CORRELATION OF STRAIN ENERGY RELAXATION WITH STRAIN PROPAGATION SECTORS

Sample No.	Vertical Planes Showing Highest Strain Energy in Relaxation		Sectors of Preferred Strain Energy Relief Under Stress	
	Primary	Secondary	Primary	Secondary
G2	East	West	West	East
G3	East	West, North equal	East	West

TABLE 6

Sample No.	Direction Line of Break Plane		Preferred Quadrant of Fracture Propagation
	*Mag.	Dip	
G2	090	85N	West
G3	104	64S	East

TABLE 7

Sample No.	Fracture Planes				Resultant Field Force Vector	
	Predicted Shear Plane		Break Plane			
	Strike °m	Dip	Strike °m	Dip	Direct. °	Inclin. °
G2	066	56N	090	85N	127	+43
	132	59NE				
G3	087	43N	104	64S	128	-48 ?
	028	65NW			308	+24
	044	68SE				
	060	50NW				

RESULTS

In a great majority of cases fracturing occurred close to prefracture determinations of planes of preferred shear or close to the strikes of a complex of shear planes with slightly different strikes. In many, the angles of dip varied, but few are contradictory within the relation of near horizontal to near vertical orientations.

Potsdam Sandstone. Seventeen fractures occurred in ten samples, including six samples with more than a primary fracture plane. Multiple fractures were predicted in all ten samples. All but two samples fractured within the NE and SW quadrant. The exceptions fractured between 321° and 342° and were secondary fractures. There was considerable variation in dip from near horizontal to near vertical and this was not correlatible with pressure or depth. The preferred direction of strain propagation was towards the north. This is a very broad coverage but it may be seen by reference to Tables 1 to 5, that all recorded times show a propagation preference for North and West, or North and East; never a preference toward South. This suggests mostly an up-dip propagation preference. From the strain relaxation rate graphs (Fig. 6) it is seen that there is no preference for the development of higher shear strains on the south faces of samples. However, this does not necessarily infer that a directional propagation preference for south could not be shown on other faces. If an internal examination could be made other characteristics might be observed.

Lyndhurst Marble. Ten fractures occurred in the nine samples tested. Only one showed more than one fracture but all displayed multiple preferred planes of shear before fracture. It was most noticeable that there were many planes of similar strike with different dips in unit samples. This feature is common among anhydrites and other recrystallized materials. Two fractures were noticeably different in strike from all others, but they agreed closely with each other. There were two distinct families of preferred shear planes.

From the recorder time base scale it was measured that strain propagation directions were in the NE and SW quadrants. It is possible that there may have been a much broader spread if the dips had been fairly flat.

In order to compare high strain energy release with fracturing directions, Table 4 has been compiled. It shows a clear trend, though not necessarily conclusive, that fracture strain propagation directions were towards the vertical planes of the prepared samples exhibiting the highest strain energy release in relaxation.

Principal Strain Directions. Five sandstone and six marble samples were able to be instrumented in three mutually orthogonal planes. The statistical principal strain directions in each plane were measured as components of a major field force. These components were plotted over the meridional stereonets to graphically measure the resultant force direction and inclination. The figures are presented in Tables 1 and 2. For the marble samples the resultant force direction lay between 130° mag. to 164° mag. and inclined at 42° to 75° to the horizontal. Corresponding characteristics for sandstone were 150° mag. to 163° mag. and inclined at 34° to 56° to the horizontal. Tables 1 to 7 show the relationships which are also shown diagrammatically in Figs. 15 to 17.

The degree of confidence levels established by a statistical analysis shows the final relationship of predicted to actual fracture planes. This is shown also diagrammatically in Figs. 18 and 19 and in Table 8. The secondary groupings are of only a very few observations, mainly secondary, and form the NW-SE sectors.

Discussion of Results

Fairly recent geological and petrofabric analyses [P. J. Clark (1959), MacLintock and Terasmai (1960)] conducted over a large area, including the sample locations, have determined the following conclusions which appear to be related to the results from this analysis, within the scope of preferred fracturing directions and major force distribution.

1. Measurements of surface stria and tension cracks show the directions of the last ice movement to be NW to SE. This could produce trajectory lines of strain at some depth and azimuth of about 145° mag. It is suggested that the field force resultant could be influenced by a horizontal component of glacier movement.
2. General uplift of land exists on the north side of the St. Lawrence Valley and depression of the south (U. S. A.) side.
3. The strike of diabase dyke intrusion is about 160°.
4. Petrofabric analysis has shown the existence at one time of a SE to SW direction of compression, producing a space lattice fabric flattening in rocks of this Grenville region. This has resulted in a regional foliation strike of about 040°. Rocks break most easily along this direction line.

Relation of Fracture Orientation with Pressure

The pressure required to fracture the unconfined samples was compared with the angles of dip of fracture planes. The values have been plotted on semilogarithmic graph paper in Figs. 20 and 21. Two granite samples have been included with the marble samples.

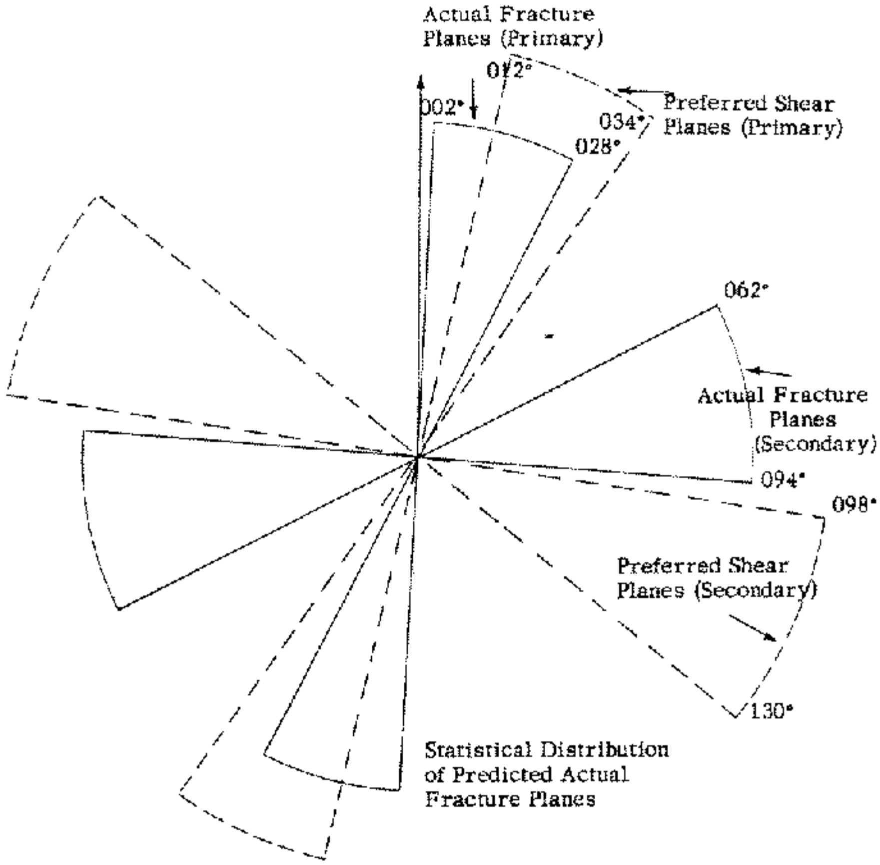


Figure 18. Potsdam Sandstone.

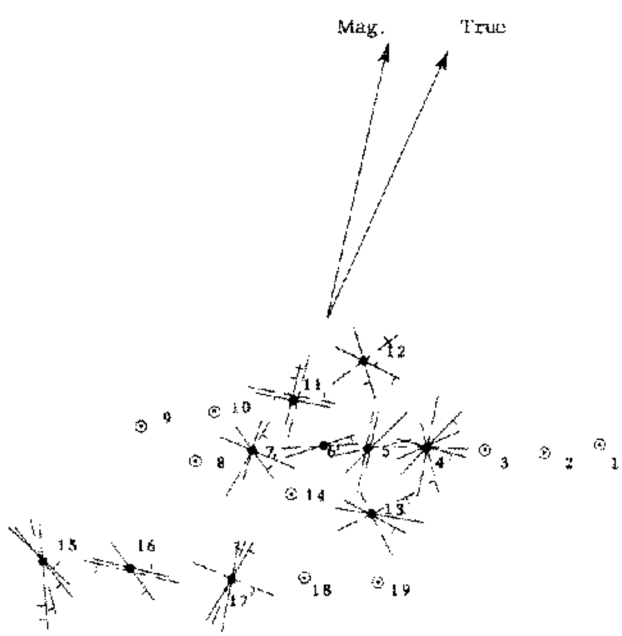


Figure 18A. Potsdam Sandstone.

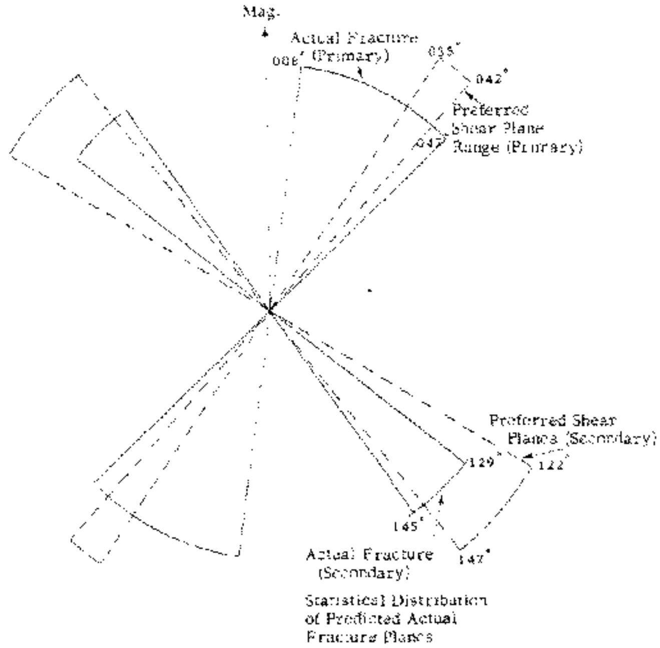


Figure 19. Lyndhurst Marble.

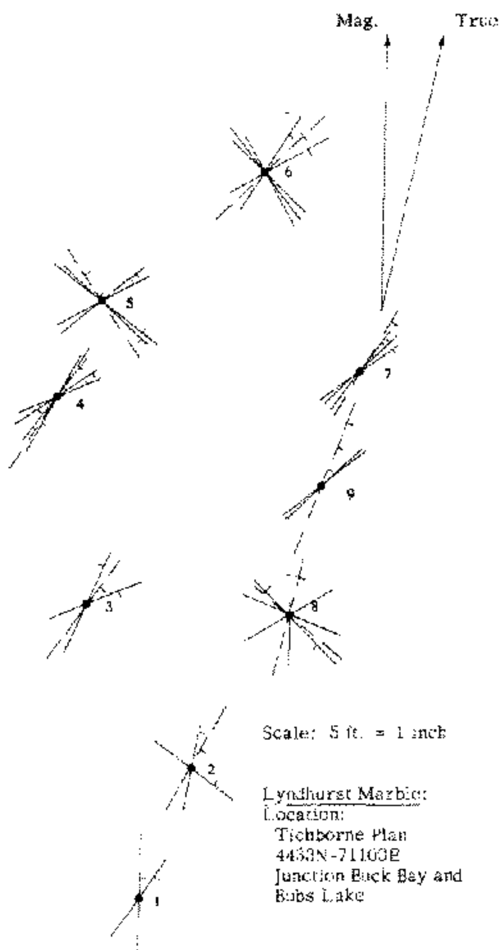


Figure 19A. Lyndhurst Marble.

TABLE 8

APPLICATION OF STUDENT "t" TEST TO DETERMINE
LIMITS OF RANGE OF TESTS
WORKING ON 95% CONFIDENCE BASIS

Samples	Group		Number of Samples (n)	Mean $\bar{x}$	t_c from tables	$S = \sqrt{\frac{\Sigma(\bar{x}-x)^2}{n-1}}$	Range $\bar{x} \pm \frac{t_c S}{\sqrt{n-1}}$
Lyndhurst Marble	Fracture Planes	1.	9	028°	2.31	23.6	028° ± 19°.3 = 008°.7 → 047°.3
		2.	2 only				129° → 145°
	Preferred Shear Planes	1.	17	039°	2.31	6.0	039° ± 3°.5 = 035°.5 → 042°.5
		2.	8	132°	2.31	11.2	132° ± 9°.8 = 122°.2 → 141°.8
Potsdam Sandstone	Fracture Planes	1.	10	015°	2.31	16.4	015° ± 12°.6 = 002°.4 → 027°.6
		2.	7	078°	2.31	17.2	078° ± 16°.2 = 061°.8 → 094°.2
	Preferred Shear	1.	13	114°	2.31	23.45	114° ± 15°.6 = 098°.4 → 129°.6
		2.	9	023°	2.31	17.85	023° ± 15°.6 = 011°.7 → 034°.3
Granite			Insufficient evidence for statistical validity				

Lyndhurst Marble. There does not appear to be any relationship between fracture planes and overburden pressure. All samples were obtained close to surface. Of 11 results, nine plotted in a pattern suggesting an exponential relationship of pressure and dip. By selecting a line drawn through a mean of the points positions obtained by plotting p.s.i. pressure in the vertical (log) y axis, and degrees of dip along the horizontal x axis, an equation, $\log y = c\,x + D$ [Wood (1954)] appears to satisfy all conditions between about 40° and 90° dip. By substituting values for x and y at two points on the line, - log pressure p.s.i. = 0.0040 (DIP°) plus 2.2068.

The substitution of various values of dip agreed well with this equation, but only between about 40° to 90°. The following explanation is offered for this characteristic.

It appears possible that at dips between 40° to 90°, at higher fracturing pressure, the material behaves primarily like a Maxwell material with Hookes Law predominating.

$$\frac{d\sigma}{dt} = \frac{\sigma}{\eta} + \frac{d\sigma}{dt} \cdot \frac{1}{E} \quad \text{(Maxwell Liquid)}$$

(40° to 90°)

While, at dips less than 40°, at lower fracturing pressure, the material essentially behaves as a Kelvin material.

(0° - 40°) $$\sigma = E\epsilon + \eta \frac{d\epsilon}{dt} \quad \text{(Kelvin Solid)}$$

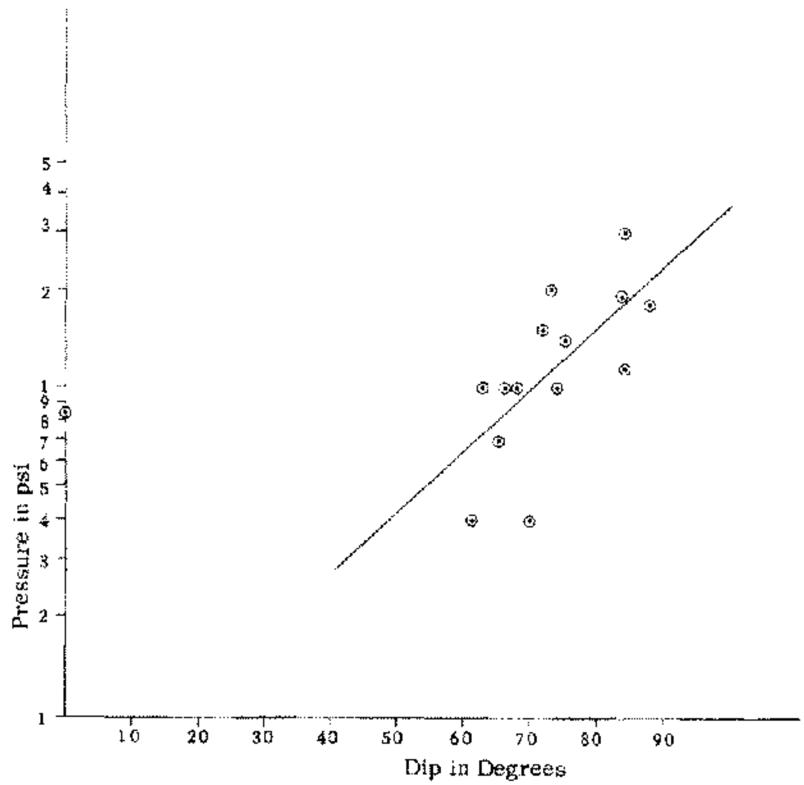


Figure 20

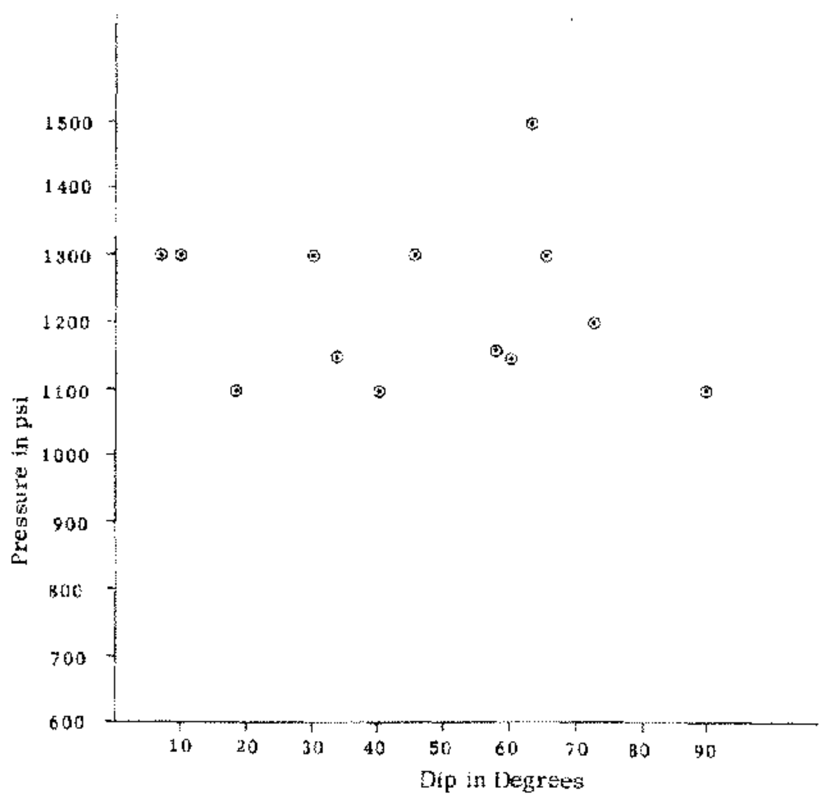


Figure 21

Therefore, at lower pressures (for longer periods of time) viscous flow and fractures of dip less than 40° would occur. Conversely, at high pressures and for short periods of time elastic rupture and angles of dip greater than 40° would occur.

For the sandstone samples no such relationship appears possible. At least, within the range of samples used and method of testing applied. A very even distribution of angle of dip with all pressures is obvious from the graph shown.

CONCLUSIONS

1. Planes of potential hydraulic fracture may be determined by a dynamic qualitative method involving a laboratory test on an unconfined rock sample. A reasonable degree of confidence of the limits may be determined by statistical analysis.
2. The degree of difficulty in predicting planes of potential fracture is inversely proportional to the magnitudes of shear strain determined in relaxation.
3. It has been demonstrated that such shear planes must be taken into account when designing a conventional mining design.[3] This research has shown that the general hypothesis is also correct for the propagation of a hydraulic fracture, where the same principles are involved, using laboratory techniques on oriented unconfined samples.
4. It has been demonstrated that planes of fracture are definable, but the actual azimuth of preferred strain propagation may vary by 180° along the shear plane.
5. The agreement in fracturing direction varies between individual samples but an average direction is definable.
6. There may be primary and secondary fractures generated in the immediate vicinity of a borehole.
7. Fracture propagation directions favour the direction defined by strain energy relief in relaxation.
8. Correlation should be looked for in geologic studies by other methods of investigation; for example, petrofabric analyses.
9. Within the scope of the examination of the unconfined samples it cannot be concluded that pressure to fracture is a function of depth. Similarly, the orientation of a fracture plane is not a function of depth. Under defined rheological conditions of rock characteristics such a feature can exist, but it can be termed by no means a general rule.

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APPENDIX

MEASUREMENT OF STRAINS

Since photoelasticity seemed to be the most likely basis for a method of measurement, work described by the following authors was reviewed. Photoelastic observations of the development of strain patterns were first recorded by Sir David Brewster in 1816. His preliminary observations were further developed by Maxwell (1850); Newman; Coker & Filon (1920); Frocht (1941). Jessop and Harris (1949) further developed the studies of photoelastic phenomena by more detailed studies of strain patterns and Frocht (1941) described, at considerable length, work in the development of isoclinic patterns. This work was continued by Zandman (1956) and Emery (1960); Pegler (1962). Zandman developed a transparent photoelastic plastic (Photostress) for use in his method, and later, Emery in 1960, developed the use of a similar material (Stress X).

Prior to 1949, no suggestion was offered that a reflected light technique could be used for the measurement of strains. It was not until Zandman's work was published in 1956 that the possibilities were realized. No work was produced before 1959 in connection with the reflective photoelastic technique of measurement of strains in rocks.

Almost all work published before 1956 that was reviewed, only produced descriptive results of a relative nature. Much may be classified as merely pictorial distributions of strain developed by applied stresses in the laboratory (Gibson, 1956).

He was quite correct in his opinion of the techniques known at that time. Further references are Phillipe & Mellinger (1957), Roux, Denkhaus & Leeman (1957), and Irving (1946). Until about 1956 all work carried out in this field was by transmitted light. The results are only useful in a limited manner.

Photoelasticity by Reflected Light

The basic principle is that some transparent materials under stress transmit only polarized light, and that double refraction occurs. The directions of polarization are identical with the directions of principal strain, and changes in the indices of refraction are linearly related to components of strain. By transmitted light, refraction components are passed out into air again from within the plastic and magnitudes of strain are measurable from the reverse side of the photoelastic plastic.

One operation only by normal incident light produces measurements of strain-difference. Two operations of measurement are necessary to measure magnitudes of strain by reflected light, using principles of oblique incident light. Each is a measure of the relative retardation of light through a doubly refractive photoelastic plastic disc of known optical constants and thickness.

Some advantages of a reflective photoelastic method of strain analysis are as follows:

1. A qualitative determination of the stress directions and changes in distribution of stress, caused by man-made equilibrium changes, such as excavations, can be made.
2. Models, in the correct sense, need not be used for the above determination, therefore, model proportional difficulties need not be considered. The prototype is exactly the same material as the model.

RETARDATION OF NORMAL INCIDENT LIGHT

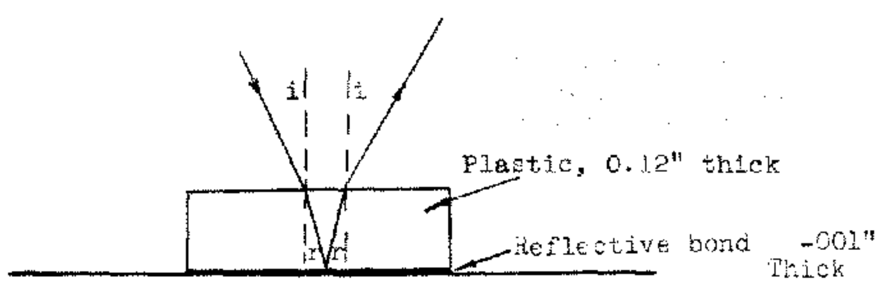

Figure 1

A beam of light, on entering the plastic is retarded and refracted. The light, on passing through a parallel plate of a stressed transparent plastic, is split into two waves whose vibrations are respectively parallel and perpendicular to the initial surface.

The retardation produced by the strain in the plastic in each wave is proportional to the stress, but the constants of proportionality are different. It is also proportional to the thickness of the plastic. One wave is retarded more than the other. The refractive index causes a bending of the beam, which, however, on striking the lower reflective surface is totally reflected, and is again refracted on passing out of the plastic into the air.

If the angle is small so that the incident beam enters the plastic very close to normal to the surface, minimum refraction occurs. The beam again being totally reflected passes out of the plastic into the air with minimum refraction.

Double refraction does occur, but only one point on the plastic surface will be seen when viewed from nearly normal to the plastic surface. The plastic is normally doubly refractive when strained by the bond to the other material. The birefringence resulting from the strain will still be observed.

From Hooke's Law --

$$E_1 - E_2 = \frac{1 + \mu}{E} (\sigma_1 - \sigma_2)$$

$$\therefore \quad E_1 = \frac{1}{E} (\sigma_1 - \mu \sigma_2)$$

$$\& \quad E_2 = \frac{1}{E} (\sigma_2 - \mu \sigma_1)$$

and $(E_1 - E_2)$ equals the final determination of observed strain-difference at any point of observation.

$(E_1 - E_2)$ = Strain-Difference as a normal incident reading and is equal to the degrees of compensation $\times \frac{650}{180}$ for "Stress X"

σ_1 and σ_2 of a rock are determined by a ratio σ_1 (plastic) $\times \frac{E \text{ (rock)}}{E \text{ (plastic)}}$ and for convenience is known as $\sigma_{1p} \times 10^2 = \sigma_{1R}$ in p. s. i., where an assumption must be made that E of a rock may be at least $10^2 \times$ E of the photoelastic plastic used.

To observe the strain pattern of rock in relaxation a photoelastic plastic is bonded directly to a freshly-cut surface. Inter- and intragranular strains are transmitted by the bonding epoxy to the photoelastic plastic. The strain is proportional to the produced birefringence change, and the changes first appear as points of higher colour order. With time the number of points increases and they often join to form families of lines of constant colour density showing shear traces on the rock surface. Plastic is bonded to orthogonal faces of a sample and the individual traces are then observed to form planes. The strikes and dips, may be measured directly from the oriented sample.

At any point of high shear strain the directions of major and minor principal strain are determined by adjustment of the polarizer axes until maximum extinction of isoclinics is observed. The major and minor principal strains are differentiated by reference to increasing or decreasing colour order, and the direction of compensation of the analyzing polarizer in an isochromatic view through the polariscope.